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Universe of Discourse and Existence

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Abstract: There is a fairly widespread belief that the problem of existence is not an essential issue for logic. Logic, though formal, must deal with the problem of existence. However, logic should be limited to describing "*formal existence*" or "*existence of a formal system*". However, the logical problem of existence and how to treat and resolve this problem differ completely from the corresponding metaphysical problem. It is possible to deduce that formal existence is nothing other than belonging to the universe of discourse, so proposing a solution to the logical problem of existence in an epistemological, rather than a metaphysical, context. In this paper, we conclude, from a formal point of view, no universe of discourse is given in advance; any universe of discourse that satisfies the necessary conditions can be used. The extended epistemological belief that there is a universe of discourse defined rigorously, which would be the true and should be "*the universe of discourse of logic*", cannot be justified.

Keywords: belief; concepts; effective existence; formal existence; meinonguian objects; nonmonotonic logic; satisfaction functions; universe of discourse

1. Introduction

From the historical point of view, the decisive attempt to solve the problem of existence for symbolic logic is due to Russell, who developed it especially in his theory of description. However, by the fact of including extra logical considerations into the formal treatment of the problem, Russell's proposal is unnecessarily complicated, losing the universality expected of a formal science.

Ostensive definition [1,2] is the process by which an individual receives instruction to understand a lexeme in a different way than through the use of other lexemes. The ostensibility of a sender is in inverse proportion to the quantity of information being provided by a particular language. An ostensive definition conveys meaning using examples. In this case a Sender refers to an absolute being or referent [3], also known as designatum [4]. Consider Russell's phrase *"The king who now governs France is bald"*. This proposition was considered a false ostensive (O) [5–7], not because the King of France has hair, but because the king who governs France now does not exist. For an ostensive proposition to be true, it is necessary that the respective subject exists and also corresponds to the predicate. The logic, though formal, must deal with the problem of existence. However, the logic should be limited to talk about "*formal existence*" or "*existence on a formal system*." That something, which formally exists, is there also really, and does not depend on formal logic, but rather on epistemology, that is, what is understood by "*real*" and if a real correspondence to the system is given and how much.

A higher system of functions is distinguished individuals, classes of individuals, relations between individuals, and classes of classes of individuals, e.g., "There are some ... or There *is* ... " can be formally defined in these systems with the words "*no*", "*or*", and "*all*", for both the individual and classes. Then, we can deduce that *formal existence* is nothing other than *belonging to the universe of*

discourse, where the so-called universe of discourse is the collection of individuals, i.e., values that meet or do not meet any function (first order) of the system. The universe of discourse is not absolute and given in advance. On the contrary, if we apply symbolic logic to any field of science, a belief system or daily life problems, we fix a universe of discourse. This fixation is arbitrary, provided that the following conditions are satisfied:

(1) The universe of discourse has at least one element.
(2) That there are no elements in different orders, such as individuals or sets.

The universe of discourse (or simply universe) is the set of entities over which certain variables of interest in some formal treatment may range. The universe of discourse is usually identified in the preliminaries, so that there is no need in the further treatment to specify each time the range of the relevant variables [8]. Once these two conditions are met, there is absolute freedom. We might consider, for example, implementing a system where the universe of discourse is formed of soccer teams. The practical value of this system would be very small because it could not deal with a problem that does not relate directly to soccer clubs. In this case, only soccer clubs exist.

However, there is nothing exceptional in working with a very limited universe of discourse. This is often done in mathematics, where the universe of discourse is restricted, for example, to natural numbers or points in the plane. Then, there are just the natural numbers or points in the plane.

Then, and as said, "*formally exist*" (denoted as $\exists_F$) acquires a relative. What exists for a system or a system application is not necessarily there for another. Usually, the universe of discourse of technology is formed so that is not perpetually mobile. However, it could be a fantastic technological universe of discourse that includes perpetual motion; for this other system, there is the perpetual mobile. For the same reason, an investigation into the mythological animals, unicorn or fairies exists, contrary to common usage in biological research.

There are not many examples about of universe of discourse in the classic logic literature. However, it is possible to find driving in its examples, which allow a negative answer to the question to be given. The universe of discourse of these examples includes what really exists (in a physical sense), and also spirit, justice, love, resignation, etc., but not Pegasus, centaurs, fairies, or dragons. The two crucial questions are:

(1) Who decides this and what are the criteria?
(2) Will it be included in the universe of discourse that really will exist with some probability?

In the affirmative case, the Siamese twin brothers who could be born on 1 January 2100 would be included. If not, the next solar eclipse will be excluded. What and where is the limit?

2. Concepts and the Problem of Existence

Existence remains itself a serious problem in the philosophy of language, metaphysics, and logic [9], and is one connected to some of the deepest and most important problems in those areas. Many of the issues can be organized around the following two questions:

(1) The existence of a property of individuals.
(2) Assuming that existence is a property of individuals, are there individuals that lack it?

Existence is a property of individuals because sentences like "*The horses exist*" are grammatical and there is a class of all individuals and hence a corresponding property of existing. There is a controversy as to whether the logical form of a sentence like "*The horses exist*" is really subject-predicate in structure and so whether the English verb 'exists' really is predicated of individuals. The question can still be raised whether existence is a property of individuals involved in our talk of what exists and what does not, which is then a question about the logical form of the sentences used in our existential discourse.

There are two sets of intuitions that seem to pull in opposite directions. The first concerns the transience and contingency of existence. Things come in and go out of existence through time.

2.1. The Theory of Meinong Pure Objects

In his theory of the object, Meinong [10] developed one of the most important theses of his thought; namely, that every object implies a pure object (*reiner Gegenstand*). A pure object means it is only an object, which is not an existing, or even ideal thing, but is involved in any other existing or ideal object. The current use of the term "object" is the thing, but this is not the meaning Meinong gave it. What definition can be given of the concept of object? According to Meinong, it is not possible to give a comprehensive definition of object: "*object*" is a primitive notion that, as such, does not allow a definition in the strict sense. That is, there is a more basic and generic concept, to which one can refer, to provide a strict definition. According to classical logic, the perfect definition is according to the genus and the *differentia*; but there is no rigorous definition through genus and specific difference, since everything is an object, everything falls under the possibility of being an object [10]. Therefore, the *genus* is as object as the *differentia*. Note that something similar happens with the notion of being in Aristotle: Being is not only a genre, but it does not fit a definition by gender and difference, because everything falls under the notion of being. Despite the similarities, at this particular point, between Meinong and Aristotle, the notion of being *qua* being is formally distinct from that of object *qua* object.

While it is not possible to give a rigorous definition of object, it is possible to make an imperfect description of objects by an etymological description, and by resorting to the experiences that lead to them. The etymology of the word "*object*" can discover interesting elements. In English, we have the word "*object*", which comes from the Latin "*obiectum*". In German, there are two words; namely, "*Objekt*" and "*Gegenstand*", which historically have been used more or less interchangeably, although Meinong distinguishes his theory of the object. The term "*Objekt*" is, as the English word "object" is obviously of Latin origin, while the term "*Gegenstand*" has Germanic roots. Surprisingly, both words have the same etymological meaning: *ob-iectum* means what is thrown or is lying (*iectum*) in front (*ob*); and in parallel, *Gegen-stand* means what is or lies (*stand*) in front (*Gegen*). In other words, the object is the converse opposed to a psychic act. According to the description of the object by the experience that leads to it, an object is anything to what a psychic act is directed, and psychic act means any act of representation, judgment, feeling, etc. This descriptive definition is a conclusion, which Meinong derives from a quite obvious principle: One cannot think without thinking something, where "*something*" is the object. It is quite impossible to know without knowing something; the object is necessary for the act of knowing, and the act of knowing requires the object, but the two are not in the same relationship. It would be better to say that the object is the "*counter-pole*" of the pole which is the act.

Meinong suggests the need for a science whose subject is precisely all the objects, a theory of objects including the objects of mental acts that are noncognitive. The objects, which intentionally constitute cognitive acts, are not only the existing objects. All that exists, including what has existed and will exist, is infinitely smaller when compared to the totality of objects. Even the ideal infinitely exceeds the extension of the existing, present, past, or future. If next to existing objects there are other non-existent objects, they must also become intentionally somehow to justify that could be subject of a true predication. Meinong thinks that the so-called nonexistent objects have been obliterated in the history of philosophy, and that its forgetfulness is a negative consequence of what he calls "*prejudice in favor of the real*." Meinong reaches this conclusion by detecting the lack of attention that most philosophers have provided to nonexistent objects, although nonexistent objects are essential to understand not only human knowledge, but also to understand logical principles as important as that of non contradiction. To state the principle of contradiction, it has to represent both the object "*that exists*" and the object "*which does not exist*" (nonexistent), otherwise it could not enunciate the most fundamental principle of philosophy. In this way, the nonexistent object enters the scene from the beginning of our knowledge of reality, and always accompanies it. A good example of what Meinong calls "*prejudice in favor of the real*" would be the notion of object proposed by Frege. For this, an object is something concrete, an existing individual, a thing with extramental existence. Frege's criterion is purely ontological, while ignoring the epistemological element. On the contrary, for Meinong,

the criterion is primarily epistemological, namely, the term being (intentional) of a psychic act. Now, it turns out that etymologically and really, the notion of object is not ontological, but epistemological: the name of the object is an intrinsically connoted real thing; that is, be-object (the *obici*) is not a real condition in the thing, but a name from outside, from the knower. Hence, it is held that to the concept of the object of Frege, is opposite to the intentional and epistemological conception of Meinong, which has in its favor the etymology of the word object, as noted earlier. Meinong does not deny the existence of things, but rather denies that one can refer to them without first their being constituted into objects: It is not possible to know something without this something being an object. In this sense, Meinong did not call himself "*realistic*", but "*objectivist*".

We have, therefore, that everything is an object, or better, everything is subject to a possible thought. The expression "*everything is an object*" does not mean "*every object is an object,*" which is a tautology, but "*all that is (thinkable), is an object*". Note that the object is different from being; It is defined as an object, when there is a relationship with a conscience. Being and object are not synonymous, nor absolutely identical, but formally distinct, although they may be physically identical. The mere fact that something is heavy is *eo ipso* an object. Just something that was not subject to a possible thought would be a thing without being simultaneously an object. Perhaps a distant star, nobody knows, would be a case of one thing that is not an object. However, this is actually impossible, because the thesis that "*everything is object to a possible thought*" is not limited to what at this point no one knows, since it also extends to what is known in the past or what will be known in the future. However, even this is also incorrect, as "*be-an-object-of-a- possible-thought*" would be valid even if it is a fact that nobody knew, knows, or can know. The possibility of what is spoken here, implies that it is possible to know, though no human knower knows this fact in the present, past, or future. Therefore, it is sufficient to qualify as an object, that which is knowable by possible knowledge. Meinong even suggests something more radical, which would be according to the following formula: Although there is no one who knows it in the present, past, or future, at least it is possible to conceive of an infinite intellect who can know. When Meinong speaks of "*a possible infinite intelligence*", he is not here asserting the existence of any infinite intelligence, but rather saying it is "*possible to conceive of an infinite intellect*." In this way, everything falls into the category of object. In this sense, the following expressions seem equivalent (although not formally identical): "*Everything is an object or can be,*" and "*everything is an object to infinite intelligence,*" because infinite intelligence is about the real past, present, and future, and something possible, that is never going to be real. Meinong now concludes that if everything is an object, then, a science of objects would be the most universal science, even more than the science of being *qua* being, Metaphysics. Being is just another object, just a real or existing object, while the object theory would have to deal with all objects, not just existing ones. The idea that Metaphysics was the most universal science would be one of the consequences of prejudice in favor of the real. Aristotle would reject this conclusion, because, although Metaphysics deals with real entities, he also deals with unreal entities. The concept of being is analogous; this means that the real being considered as a primary analogate, and the unreal in reference to the real entity. Later, the metaphysicians debugged this idea by saying that Metaphysics deals with the real entity and unreal *ad instar entes*; that is, as if to emulate a real entity. However, Meinong is right to indicate that the science of the object is universal, and although it is problematic that it is more universal than Metaphysics, it has to give credit to Meinong, because in classical metaphysics, Aristotle must include the object of metaphysics, transcendental of being, among which is the true transcendental. The being as true is the entity referred to intelligence, and at this point, can be established as an interesting parallel with the theory of the object.

Meinong mentions four types of objects (*Gegenstände*) as the fundamental types of intentional acts. Thus, there are representation objects, objects of judgment, emotion, and desire; namely, object (*Objekt*), objective (*Objektiv*), dignitativo (*Dignitativ*) and desiderative (*Desiderativ*), respectively. In this quadripartition, there are two kinds of objects that deserve our attention; namely, the object (*Objekt*) or object of representation, and the objective (*Objektiv*) or object of judgment. Meinong reserves the term

Gegenstand to refer to objects in general and reserves the term *Objekt* to refer to the special case of the objects of representation. An object of representation is a number, a shape, a color, a tree, etc. Unlike representation objects, all goals are ideal and never real, while objects of representation (*Objekte*) can be real or ideal. There are ideal objects that exist alongside existing, such as the difference between red and green. The object difference is an object that is based on two objects; for example, red and green. The difference is an object of higher order, or simply a *superiora*, while red and green are objects of lower order, or simply *inferiora*. What is striking is that, while the red and green objects exist, the difference, which is based on them, is there itself, but has an ideal being, which Meinong called "subsist" (*bestehen*). The difference does not exist between the red and green objects, but subsists (it has an ideal being) between them.

Number, for example, is a super ordinate object that is *inferiora* to numbered. The being of the number, such as the difference is not, therefore, existing but subsists (*bestehen*), which is the essence of ideal objects. Meinong only considers the *numerus numerans*, not the *numerus numeratus*. The objects of higher order, the *superiora*, are objects that "subsist" on objects of lower order or *inferiora*, which exist (*existieren*). Not all objects of higher order are based on existing objects, but at the end of the series, find an existing object. For example, the difference between two numbers has *inferiora* to two numbers, and these in turn are superordinate objects that can have as *inferiora* real objects. Difference and number are objects of representation, but this does not mean that all objects of higher order are objects of representation, as also the object (*Objektiv*) is always an object of higher order. The objective object of judgment (*Objektiv*) the fulfills the conditions of a superordinate object. An objective is, for example, the following true judgment, "*an airliner flies through the sky*". With this judgment we not only know the flight of an airplane, but also that there is a peculiar object (or, more precisely, what Meinong called "objective") by which I know my judgment is true. This object of judgment is neither the airliner, nor heaven, but the goal "*an airliner flies through the sky*". The object "*airliner*" is not judged, but the objective is that "*an airliner flies through the sky*". This objective is completely different for elements that it comprises, and this can be easily seen if it is noticed that the real object the plane flies through the sky, and even the real objects passengers actually fly through the sky, but "*an airliner flies through the sky*" turn and not fly through the sky, or make any physical activity. If the objective, "*an airliner flies through the sky*", were false, it would be because there is another target, "*an airliner not fly through the sky*", that is judged as true, and not because the object of representation, "*airliner*", which is an integral part of the objective, is the same object for representation appears in both a true and a false judgment. Similarly, if I judge that "*the unicorn does not exist*", the judgment of this objective is true not because the object (for representation) unicorn does not exist, but because of the objective, "*the unicorn does not exist*", which is once there, but subsists. The same could be said of the objective "*there are Airliners*", which affirms the existence of these aircraft; but, the objective itself does not exist, but subsists. If the target, "*there are Airliners*", is there simply because its constituent parts are there (here, *Airliners*), then by the same reasoning, the objective to include the first objective—"'*There are Airliners' exists*"—also exist, and so on *ad infinitum*, creating an endless chain of real existences and effective by the mere fact of judging the existence of something. To avoid this undesirable *regresum ad infinitum*, Meinong maintains that an objective cannot exist, but has an ideal being, and subsists (*bestehen*).

We can make true judgments about objects that exist, about objects that do not exist, and what may seem paradoxical, about objects that are inherently contradictory. Regarding the latter, we can form a true judgment about a contradiction to say that the contradictory does not exist. For example, we can make true judgments about perpetual mobility, which does not exist, cannot even exist. We can say that we are now thinking about the perpetual mobility, which is impossible. The impossible is not the act of judging, but the object, "*perpetual mobility*", since the psychic act is real, and nothing real can be impossible. Additionally, as any intentional act has its purpose, then, the act of thinking about perpetual mobility will have perpetual mobility as its object. One can say that "*perpetual mobility is impossible,*" and this is a necessary truth, or we can also say that "*perpetual mobility cannot exist*", which is also a necessary truth, and we can even make judgments like "*perpetual mobility is perpetual* ",

which is an analytic truth because the predicate is included in the subject. The simple fact that one distinguishes an impossible object of another (for example, *the perpetual mobility of gas*), or we can say necessary and analytical truths, or impossible objects, it is enough to say that what makes the subject (the impossible object) must be represented somehow.

Meinong gives a more radical step. There are truths that remain even or never were, nor were objects of thought (but can be thought). Such objects have at least the characteristic of "*not being thought of by anyone*." In this way, although no one thought of perpetual mobility, the objective "*perpetual mobility does not exist*" would be even necessarily true. To be the object, there is no need to exist or even to subsist. Hence, the apparent paradox of the Meinonguian formula: "*There are objects about which it is true to say that there are no such objects,*" that is, there are objects that do not exist. Meinong says we can say that there are objects that have existence, others with subsistence, and eventually other objects that have neither existence nor subsistence, so there have to be some, as in the case of impossible objects, which neither exist nor subsist. The independence of being of the object concerning its nature is such that, to know whether an object exists or not, before one has had to think, without including their being. Even to know about an object, its being does not enter into the consideration of that object. One can analyze the nature of a rock, a bird, or a planet without having to indicate its being. From here, Meinong establishes the important thesis of the independence of the essence (*Sosein*) in respect of the being (*Sein*) of an object. This principle states that the essence of an object is affected neither by existence nor nonexistence, or nonbeing (*Nichtsein*). Their being (and its nonbeing) is entirely extrinsic to its essence. Therefore, the fact that there is perpetual mobility is not related to the fact that perpetual mobility has the essential properties of being mobile and perpetual. In this sense, one can say about any object (*Objekt*) that its essence does not exist, or more correctly, it must be said that both existence and nonexistence is extrinsic to the nature of the object. The essence (*Sosein*) of the object is what Meinong called "*pure object*", whose status is being beyond being (*ausserseind*); that is, a pure object remains beyond being and non-being.

The principle of independence applies only to objects of representation and not properly objects of judgment or objectives. While the object's representation (*Objekt*) may exist or not exist, it may subsist or not subsist or may "have" only *aussersein* (the state beyond being); the objective, however, always has being. While the object of representation does not have any being, and therefore is simply a pure object, it will always be the case that both objectives "*A exists*" and "*A not exist*" have their own being, subsistence. All objective things subsist, and therefore, are unable to have either existence or the mere state of *aussersein*. What Meinong called subsists for an objective is its truth, it is true or false being. If it is said that "*the unicorn does not exist*", this objective subsists as true "being", and if they say wrongly that "*the unicorn exists*", this goal also subsists as false "being". That is, the truth "being", or subsistence of the objective, is something that belongs to him and is inadmissible.

Meinong develops the theory that it is possible to form objectives that still have not been decided whether they are true or false. For example, "*the author of El Lazarilllo de Tormes lived in Castilla*." Since the author is anonymous and little is known about him, the judgment on its truth or falsity is suspended but not canceled. Meinong says such objectives are simply assumed, whose being also subsists, that is, have a truth being and that either is false or true, although we do not know, or what is the same, has a veritable "being", which may (or holds the possibility to) be affirmed or denied. The suspension of judgment does not mean that there is a possible judgment, which is in itself true or false. The act of judging has to take an objective, it is a mental act, while subsisting or the truth being of the objective, is an element *a parte objecti*. One must maintain the distinction between what belongs to the intended object, on the one hand, and an intentional act, on the other. The objective "*five is a prime number*" is not only true, but also necessary. The true being of the objective is the subsist (*bestehen*) of a whole, where the parties are the objects of representation "*five*" and "*prime number*". If the whole has being or subsists, then it seems as should be expected that the party "five" must have being or subsist too. In this way, both subsist in this objective, as the subject "five". The logicist argument the being of the objective has been transferred, to the being of its parts. The argument is based on the

idea that if everything has being, then necessarily, the component parts have to have being. Meinong maintains that the number five certainly subsists, but not because it receives the subsistence in an objective. The logicist argument seems to be carried away by whole-part analogy where the being of everything is the essence of the part. In this way, since the objective always has a being (its truth being), the component parts always have, at least the being as subsistence, if they do not exist. The objective "*the three is greater than the two*" subsists, and component parts (objects three and two) also subsist. On the other hand, the objective "*red is different from green*" also subsists, but the parties (the red and green objects) not only subsist, but exist. Hence the logicist concludes that the constituent objects of an objective (which is the whole) always have being either as subsistence or existence. In this example, the objects "three" and "two" subsist, but not because they belong to an objective, such as logicist fallacy presupposes.

Meinong rejects such arguments, based on the analogy which considers it inadequate to analyze the objective. The reason is based on the discovery of impossible objects that cannot have any kind of being or existence or subsistence. The objective "*the round square is round*" is not only true, but apodictic. This objective subsists (*bestand*) or has truth being. If we use the argument of whole-part analogy, then the object which is a subject in this objective should have being. However, nothing impossible may have being. The impossibility is the total exclusion of being; otherwise the impossibility would not be impossible, but possibility, which is precisely rejected. Additionally, because the impossible object cannot have any kind of being, and their status is only of beyond being (*aussersein*), then, it cannot be applied to the whole-part analogy to the objectives.

Meinong justifies this as follows: If any object X constituent of an objective must have a being, then the objective "*X is not*" that subsists, denies the being (existence or subsistence) of object X. But if the whole requires a being of the parts, then in the objective "*A is not*", a being should be attributed to the object A, a different being from existence or subsistence, perhaps a third kind of being. This third type of being, then, corresponds to the entire object qua object, and every object, would have this being, even when it was denied by the being. This third type of being would be so peculiar that it would not oppose any non-being of the same class, as not being so would fall in an undesirable return *ad infinitum*. This hypothetical and problematic third type of being that neither exists nor subsists, and which has no opposition, whereas not being does exist and subsist it is simple and simply unsustainable, if the notion of being is not abandoned completely. Therefore, Meinong concludes that the state of an object where neither being nor non-being are opposites, is a state of indifference to being and non-being, or a state that is beyond being and non-being, and Meinong prefers to call it "*aussersein*", "*being beyond being*", which is a state of the object that does not include and not exclude being, or its opposite, non-being.

Suitably, all opposition between being and non-being is a matter for the objective and not the object of representation (*Objekt*) that may be part of the objective. Additionally, considering that being of the objective is not in any way referred generally to be the constituent object (as the whole-part analogy does not work with the objective-object constituent relationship) becomes, then, the objects as such (pure objects) are not essentially involved, as neither being nor non-being. The objective is about being or non-being, while the object of representation (*Objekt*) is just and properly about the essence of the object. In this way, the acquisition of an existing object always implies a representation of the existing object, which is essentially presented, and implies an objective, by which is predicated its existence. The above considerations do not mean that the object is not, nor will ever be, for being and non-being are contingent to the essence of the object; since there are objects that exist, others subsist, and other impossible objects that necessarily do not exist. That is, there are objects that carry the guarantee of non-existence or subsist only analyzing their essences. However, the question of an object (*Objekt*) exists, subsists, or may not exist, is not a matter of the object (*Objekt*), but of the objective where being or non-being is attributed. The object as an object is beyond the being; their status is *aussersein*, extra-being, outside of being. The object is by its very nature, outside to being, although one of the two objectives (which attributes the being or non-being) is necessary.

What Meinong called "*pure object*", the object (*Objekt*) as such, or object accurately considered; that is, the essence of the object as it is captured by the representation. In this sense, the pure object "has" *aussersein*, which is the state beyond being, or rather the pure state beyond being and non-being. Hence, the Meinonguian important thesis: *every object representation, by the mere fact of being subject has at least aussersein*. The problem that arises now is how it is possible that an existing object has *aussersein*? It seems clear (according to the principle of non-contradiction) that either it exists or does not exist, but may not be beyond existence and not exist as required by the *aussersein* pure state. Moreover, the principle of non-contradiction implies the law of the excluded middle, which is what the doctrine of *aussersein* seems to deny.

In the doctrine of pure object and its status beyond being (*aussersein*), which Meinong means is that the essence of the object is not any being, either extramental or mental, even non-being, since attribution of being and non-being is extra-essential, not a note of the essence. As has been done on the principle of independence, we believe that Meinong makes it clear that the essence (*Sosein*) of the object is in itself beyond being and non-being. Hence, given the distinction of being (*Sein*) and essence (*Sosein*), the essence is not being nor its lack of being, but is capable of it; even more, every essence has or has not being, but the essence in itself, is extrinsic to being. The essence (*Sosein*) is not only outside to be like existence and subsistence, but also due to be intentional. For example, in the objective "*the unicorn does not exist*", the object (*Objekt*), the unicorn, is certainly nonexistent, and in the description of the essence of this object, does not include any reference to an act of representation. When the essence of this object (*Objekt*) defined "unicornit is not included in any way "the intentional being" (their represented-being), which Meinong prefers to call "pseudo-existence" (to avoid the term "intentional existence" that easily lends ontological errors). This is based on the fact that, to judge that an object does not exist, before it has to represent, and the representation of that object cannot include the being, since the attribution of being or non-being is a matter for the objective and not the object (*Objekt*) for representation. In other words, the only thing that makes the representation is to capture the essence of the object, but not its being. The being is attributed solely to the objective. Without the objective, it could not know if an object (*Objekt*) has or has not being, because it is in the context of the objective where it is judged that something has being or not. In this sense, pseudo existence is not in a different position, which being a type of being, must belong to the objective attribute of an object. Meinong explicitly stated, that for the nonexistence, the object needs even less, if possible, to be represented, as for existence, and even to the extent that it came to be represented, could only become, at most, the existence in the representation and hence the pseudo-existence. Hence, we have that the essence of an object, the pure object, does not imply either existence or subsistence, not even its represented-being, the intentional being or pseudo-existence. This does not mean that any object, in order to be that object, must be represented, but this attribution is achieved only when forming part of an objective. The pure object is beyond being and non-being, beyond the existing and the pseudo-existence, beyond extramental and mental being.

The object (*Objekt*) can be found in four states: Existence, subsistence, pseudo-existence, and beyond being. An object can exist (this butterfly), can subsist (number five), can pseudo-exist (the tree as being imagined by me now), or only have the state beyond being (a rounded square). The interesting thing about these four states of the object is that there is a hierarchy among them. Any object (*Objekt*) that there is has to subsist and have the state beyond being, but not vice versa. Any object (*Objekt*) that exists has to have the state beyond being, but not vice versa. Additionally, every object (*Objekt*) that neither exists nor subsists (as the Impossible) has at least the status beyond being (*aussersein*). This state is the state of pure object in its pure state, beyond being, which is always present in every object, and have being as existence, and have it as subsistence, or simply do not have any kind of being. Every object, by the mere fact of being an object, has *aussersein*. For example, the object (*Objekt*) tree is an extramental object, which has existence. However, this same object's being, considered by the representation and what can make it the object of reflection, is mental, or more accurately, the object is considered only intentionally, whose being is pseudo-existence, pure and simple being-represented,

whereby the tree object is an object-of-representation. All existing objects in its direct consideration, is to be like existence; but in its reflective consideration, it has to be like pseudo-existence. The same must be said of all subsisting objects (for example, number five), which in their direct consideration have being as subsistence, but in their reflective consideration have to be like pseudo-existence. Additionally, no matter that it considers directly or reflectively its extramental or mental being, there is a fourth state of the object that is independent of these three states, it is the pure state of the object, in any consideration of being. The essence of the object tree has extramental existence, and has another mental existence or pseudo-existence when it is represented; but, as such, the tree essence can be described without reference to being, namely, if it exists or only is represented (pseudo-existence).

When a botanist describes a species of tree, for nothing enters the description of the object that studies the ontological consideration of their existence, or epistemological consideration of its pseudo-existence; simply considers the essence of the tree as if in a pure or absolute state, beyond being or not being. All notes that can be predicated of the object in its pure state are still valid when it is attributed with extramental being or merely considered, in the reflective state of our consciousness, as an object that is representing (pseudo-existence). To describe the biological essence of the tree is a completely superfluous inclusion of being as existence in their notes as well as its represented-being. The biological essence of the tree is the same, whether it has existence as if it does not, whether it is thought (and then has a being in the representation or pseudo-existence) as if it is not. Nothing is added to the biological essence of the tree, by being represented with as existence.

2.2. Russell's Objection

The notion of Meinong objects has been rejected by many philosophers, especially within the so-called analytic school. In fact, it has not just been rejected, but it has been misunderstood and the notion of a misconstrued object was rejected, which Meinong never held. One of these philosophers, once a follower and admirer of Meinong, was Bertrand Russell, who presented his alternative to the Meinonguian thesis in his work *On Denoting* [1]. Here, we just want to indicate that a notion that such thing as Russell, involve unsavory problems in the theory of predication. Russell states that object and thing are identical and interchangeable without justification; that is, he takes it as a starting point for his alternative theory that all that exists has the same extent as the totality of what is an object, and propositions such as "*Hamlet does not exist*" or "*perpetual mobility is impossible*", which are both true, would make no sense whatsoever. However, this uncritical identification between object and thing also affects the same Principle of Non Contradiction (both of which Russell defends), saying that "*it is impossible that something be and not be in the same direction*," he was thinking of something (a object) that is a contradiction (as it is and is not at the same time and in the same direction), then a predicate that it is impossible to really give; or as Meinong could say that an object that is and is not in the same sense, it is impossible to exist, but this does not detract from the fact that it can be a simple "counter-position" a real object (impossible) of the act of thought: The self-object, *the obici*, the opposed, that is not in itself any being, and at this point Meinong seems to be right.

Russell would object that the concept formed by the state of things is that there is an x such that x is perpetual and that same x is not perpetual is a false proposition of existence, and therefore a pseudo-concept, nonsense, or better, a mere *flatus vocis*. However, this is wrong; Russell does not seem to distinguish between nonsense, something that contradicts the Second Law of Thermodynamics (perpetual mobile) and a mere *flatus vocis*, such as "*hula hula ug ug*", which are mere sounds without intelligibility, while the nonsense "*perpetual mobility*" is understood, probably by all, and distinguished from other nonsense. Who understands the senselessness "*perpetual mobility*" knows that it is not a "*solid gas*". Russell is believed to form propositions such as "*perpetual mobility is perpetual*" need not represent the nonexistent object "*perpetual mobility*" as the subject of a proposition.

Russell argues for a distinction between subject and predicate, which is a grammatical structure and superficial, and the logical structure underlying grammar, in which the notion of subject is not required, but only the predicate. In this way, believe that the proposition "*perpetual mobility is perpetual*"

involves thinking about the subject "*perpetual mobility*" would be carried away in a grammatical, superficial, and misleading way, making us believe that there is a real subject, object of thought. For Russell, the proposition "*perpetual mobility is perpetual*" consists of three propositions, or better, is the synthesis of three conditions:

(1) *Existential*: There is at least one individual who is perpetual mobile.
(2) *Uniqueness*: there is only one individual who is perpetual mobile.
(3) *Conditional*: if something is perpetual mobile, then it is perpetual.

We cannot stop in the highly problematic first and second conditions, which involve an uncritical ontology ([11], pp. 33 ff., 46 ff., and 56 ff.) but we can indicate the opinion of Meinong regarding the third condition, which seems more acceptable in many circles of symbolic logic. According to the latter condition, which expresses the profound form of purely grammatical structure of subject-predicate, universal propositions of the type indicated must be converted into conditionals. Meinong objected to Russell's hypothesis that reduction to the conditional of universal propositions, substantially changes the meaning of propositions. In other words, to illustrate what Meinong has in mind, the description of the subject-predicate proposition to consciousness is lived, from a phenomenological point of view, in one way completely different from a conditional. This can be supplemented with the objection that the propositions of subject-predicate structure are propositions because they are liable to have a truth value (they are true or false), while conditionals are not strictly propositions and have no value of truth or falsity.

2.3. Linsky and Zalta's View

Thus, it seems, different things exist at different times. Likewise, of the things that in fact exist, some of them might not have existed, and different things—things that in fact do not exist—might have existed instead or in addition. The same can be said for spatial existence. Kangaroos existed in the Middle Ages, but they were not known to Europeans before the discovery of Australia.

The thesis that absolutely is actually everything and how an object is *simpliciter* is how it actually is, unactualized possibilities for an object being in some sense hypothetical ways of being for that object.

We can regiment the contingency of intuition about existence as follows, letting A be the actuality operator, where AN is true with respect to a world W under an interpretation I just in case N is true with respect to the distinguished world of I: $\Diamond\exists N(x)\neg \mathrm{A}\exists N(y)$ $(N(y) = N(x))$.

One way to substantiate this worry is to invoke Barcan's Formula, that is the mixing of axioms proposed for modal operators and quantifiers in Ruth Barcan Marcus's groundbreaking work in quantified modal logic [12] according to which all instances of the sentence $\Diamond\exists xN(x)\rightarrow\exists x\Diamond N(x)$ are logically true.

The Barcan formula is: $\forall x_{\Box}F(x) \rightarrow {}_{\Box}\forall xF(x)$ where $\Box$ is the operator of necessity. The schema reads: If everything is necessarily F, then it is necessary that everything is F. It is equivalent to $\Diamond\exists xF(x) \rightarrow \exists xF(x)$ where $\Diamond$ is the operator of possibility. The Barcan formula has generated some controversy because—in terms of possible world semantics—it implies that all objects, which exist in any possible world (accessible to the actual world), exist in the actual world, i.e., that domains cannot grow when one moves to accessible worlds. This thesis is sometimes known as actualism—i.e., that there are no merely possible individuals. There is some debate as to the informal interpretation of the Barcan formula and its converse.

There is, however, a second line of argument that does not rest on the validity of the Barcan Formula, relying instead on combining standard truth definitions for quantified and modal sentences in the most straightforward way. A solution rests on the distinction between being, in the sense of being a member of the most inclusive domain of quantification and discourse, and existing and the claim that there are objects that do not exist.

Linsky and Zalta's view [13] requires that *concreteness* is an accidental property. The self-same individual that is not concrete could have been concrete and the self-same individual that is concrete

could have been not concrete. To see this, it emerged from the modal problem of contingent existents to its temporal analogy, the problem of temporary existents. Intuitively things come in and go out of existence; what exists at one time does not exist at another. The temporal analog of Linsky and Zalta's view of contingent existents entails that everything always exists. What there is and what exists at one time is the same as what is and what exists at any other time; the domain of quantification is fixed across all times. What varies from time to time is which of those individuals are concrete. This view requires that a thing can survive the change from being not concrete to concrete, intuitive generation, and the change from being concrete to not concrete, intuitive destruction. On this view, then, seeming generation and destruction or substantial change are really forms of qualitative change, a change in the quality of concreteness. This runs contrary to the common conception that concreteness is necessary and eternal to any object that instantiates it, the divide between concrete and not concrete individuals marking a divide between categories of being that an individual cannot migrate across.

3. Logics AND Mathematical Formalization

Axiom 1. *The concept* N(x) *of a being x is attached to the existence of said being* $\exists x$.

Therefore we can distinguish two basic types of existence of an object x:

(1) *No effective (no real) existence* can be distinguished: Fictional existence corresponding to literary characters and mythical existence corresponding to beings created for systems of religious beliefs, folklore, archetypes, etc. We will use the operator $\exists_M$ for that kind of existence (existence in mind).
(2) *Effective (real) existence*, corresponding to all objects in physical reality, i.e., material and energy beings. We will use the operator $\exists_R$ for that kind of existence.
(3) *Formal existence*, corresponding to all real and not real objects. We will use the operator $\exists_F$ for that kind of existence.

The borders between the two types of stock are not clear. An object x can have two classes of existence. An apple can be a real biological object, but also a mythical object: The apple from the Tree of Good and Evil, the golden apples of the Hesperides, etc. Then, if x is an apple, then $\exists_R x \wedge \exists_M x \Rightarrow N_R(x) \underset{S}{\wedge} N_M(x)$

Axiom 2. *Real existence is included in formal existence.*

We use special symbols called *descriptions* and they are of the form $|xF(x)$, and can be read as: *The x that satisfies the function F.*

Example 1. *Consider the sentence: "The fairy". We can read it as: "the x(woman) that satisfies the function F be a fairy".*

The descriptions do not denote anything directly and can be used in sentences spoken of something existing in the physical sense, as in sentences spoken about something nonexistent in the physical sense or something whose existence is not assured.

3.1. Operations

Let Φ be a set of satisfaction functions and let F, G, and H be three satisfaction functions such that $F, G, H \in \Phi$. The proposed operations are:

(a) Conjunction: $|xF(x) \wedge |xG(x)$. Conjunction has the following properties:

Closure: $\forall F, G \in \Phi \Rightarrow (|xF(x) \wedge |xG(x)) \in |x\Phi(x)$

Associativity: $|xF(x) \wedge (|xG(x) \wedge |xH(x)) = (|xF(x) \wedge |xG(x)) \wedge |xH(x)$

Identity element: There exists an element E in Φ, such that for every element F_i, $i = 1, 2, \ldots, n$ in Φ, the equation $|xN(x) \wedge |xF_i(x) = |xF_i(x) \wedge |xE(x) = |xF_i(x)$ holds.

Inverse element: $\forall F_i \in \Phi$ there exists an element $\exists \neg F_i \in \Phi$ such that $|xF_i(x) \wedge |x\neg F_i(x) = |x\neg F_i(x) \wedge |xF_i(x) = |xE(x)$ where $|xE(x)$ is the identity element.

Commutativity: The result of the operation of conjunction does not depend on the order of the satisfaction functions. In other words, the result of combining element $|xF(x)$ with element $xG(x)$ has the same result as combining element $|xG(x)$ with element $|xF(x)$; the equation $|xF(x) \wedge |xG(x) = |xG(x) \wedge |xF(x)$ is always true.

A conjunction operation of satisfaction functions has the algebraic structure of the group.

(b) Disjunction: $|xF(x) \vee |xG(x)$. Disjunction has the following properties:

Closure: $\forall F, G \in \Phi \Rightarrow (|xF(x) \vee |xG(x)) \in |x\Phi(x)$

Associativity: $|xF(x) \vee (|xG(x) \vee |xH(x)) = (|xF(x) \vee |xG(x)) \vee |xH(x)$

Commutativity: The result of the operation of conjunction does not depend on the order of the satisfaction functions. In other words, the result of combining element $|xF(x)$ with element $|xG(x)$ has the same result as combining element $|xG(x)$ with element $|xF(x)$; the equation

$$|xF(x) \vee |xG(x) = |xG(x) \vee |xF(x)$$

is always true.

A disjunction operation of satisfaction functions has the algebraic structure of the commutative monoid.

Distributivity:
$$|xF(x) \vee (|xG(x) \wedge |xH(x)) = (|xF(x) \vee |xG(x)) \wedge (|xF(x) \vee |xH(x))$$
$$|xF(x) \wedge (|xG(x) \vee |xH(x)) = (|xF(x) \wedge |xG(x)) \vee (|xF(x) \wedge |xH(x))$$

3.2. Axioms and Theorems

There are the following axioms:

Axiom 3. *The x that satisfies F exists formally:* $\exists_F |x(F(x))$.

Axiom 4. *The x that satisfies F do not exist formally:* $\neg\exists_F |x(F(x))$.

Axiom 5. *The x that satisfies F satisfies G:* $G|x(F(x))$.

Axiom 6. *It is not the case that the x that satisfies F satisfies G:* $\neg G[|x(F(x))]$.

Axiom 7. *The x that satisfies F does not satisfy G:* $\neg G|x(F(x))$

Example 2. *Consider the sentence: The fairy is spinning gold with a distaff. The five classes of sentences will be:*

(1) The x that satisfies the function of being a fairy exists formally.
(2) The x that satisfies the function of being a fairy does not formally exist.
(3) The x that satisfies the function of being a fairy *satisfies* the function of gold spun on a spinning wheel.
(4) It is not the case that the x that satisfies the function of being a fairy satisfies the function of spinning gold in a spinning wheel.
(5) The x that satisfies the function of being a fairy does not satisfy the function of gold spun on a spinning wheel.

In these sentences, the part where the descriptions are provided, there is no reference to individuals but only to functions (as F and G) and operations. The latter appear to give explicit definitions of the sentences referred.

Theorem 1. *If* $|x(F(x))$ *satisfies a function G, then exists formally.* $G|x(F(x)) \Rightarrow \exists_F x(F(x))$

Proof. For Axiom 6, $|x(F(x))$ exists formally. □

Example 3. *If the fairy satisfies the function of spinning gold with a wheel, then the fairy exists formally.*

To prevent pseudo paradoxes, take a precaution: The expression "*not formally exist*" is not a function; it is a grammatical predicate, not a logical predicate (Kant says that "being" is not a "real predicate"). Without this precaution, one would have for example: "*The fairy does not formally exist*." However, according to Theorem 1, "*the fairy formally exists*".

Theorem 2. *If it is not the case that* $|x(F(x))$ *satisfies G, then* $|x(F(x))$ *does not exist or does not formally satisfy the function G.* $\neg G[|x(F(x))] \Rightarrow \{\neg\exists_F|x(F(x))\} \vee \{\neg G|x(F(x))\}$.

Proof.

(a) For Axiom 6, it is not the case that x satisfies F satisfies G. For Theorem 1, if x does not satisfie G, it does not exist formally.
(b) For Axiom 7 if x satisfies F, it does not satisfy G.

□

Example 4. *If this is not the case that the King of France is bald, then the king of France does not formally exist, or the King of France is not bald. We have always in bivalued logic with the choice between "the king of France is bald" and "is not the case that the King of France is bald"; but not the alternative between "the king of France is bald" and "the king of France is not bald." In the latter case the two sentences may be false for the simple reason that the king of France does not exist.*

Note 1: These theorems indicate how to deal objects that, once selected, the universe of discourse belongs or does not belong to it.

3.3. Nonmonotonicity

Nonmonotonic reasoning is especially appropriate when the knowledge is incomplete, the universe of discourse is changing, and assumptions are temporary. Nonmonotonic logics are systems in which the addition of new theorems can invalidate old theorems [14–17].

Let A_R and A_F be the two sets of axioms of real and formal systems, respectively, and Th(A_R), Th(A_F) denote theorems that are sets of A_R and A_F, respectively. Since, for consequence 2, $A_R \subseteq A_F$ and $Th(A_R) \subseteq Th(A_F)$.

Let M be an operator whose meaning is: *Consistent with the theory*.

Example 5. *x is a woman who does not have magical powers. The woman who is not able to have magical powers is described as follows:*

$$(\forall x)\left(woman(x) \wedge M\ \ has\ \ not\ \ magical\ \ powers \rightarrow has\ \ not\ magical\ \ powers(x)\right)$$

Or, informally, "for each x, if x is a woman and it is consistent to assert that x does not have magical powers, then x does not have magical powers".

A single nonmonotonic inference rule is *MP derivable if* $\neg P$ *is not derivable*.

As Moore [18] says, there are two different forms of nonmonotonic reasoning: Autoepistemic and default reasoning. The preceding Example 6 is an example of default reasoning: In the absence of

information to the contrary, the tentative conclusion is that a specific woman, a fairy, has not magical powers. However, "*M has not magical powers*" may be interpreted differently: The only women that have magical powers are the ones that can be inferred to have magical powers. Thus, the inference rule for M is now "*MP is derivable if* $\neg P$ *is not derivable*". This type of reasoning is not a form of default reasoning. Instead, it is reasoning about one's own knowledge or belief. Autoepistemic reasoning differs from default reasoning because the latter is defeasible, while the former is not.

The syntax of autoepistemic logic extends that of propositional logic by a modal operator K indicating knowledge: If F is a formula, KF indicates that F is known. As a result, K$\neg F$ indicates that $\neg F$ is known and $\neg$KF indicates that F is not known. This syntax is used for allowing reasoning based on the knowledge of facts. If $\neg KF \rightarrow \neg F$ means that F is assumed false if it is not known to be true. This is a form of negation as failure.

Example 6. *Let* U_M *be a mythical universe of discourse and* U_R *be a "real" universe of discourse such that* $U_R \subset U_M$. Let X be a set of women. Following Moore [18], we consider two theories T_1 and T_2.

Theory 1 (T_1) $T_1 \in U_R$.

$AT_1$1 (Axiom 1 of Theory 1): If x(Fairy) $/ Fairy \in X$ and $AT_2$2 (Axiom 2 of Theory 1):

$$(\forall x)\left(woman(x) \wedge Mhas\ not\ magical\ powers \rightarrow has\ not\ magical\ powers(x)\right)$$

i.e., *has not magical powers* (x) is consistent with that Theory 1. Then: $Th_1$1 (Theorem 1 of Theory 1): *has a not magical power* (Fairy).

Theory 2 (T_2) $T_2 \in U_M$.

$AT_2$1 (Axiom 1 of Theory 2): If x(Fairy) $/ Fairy \in X$ and $AT_2$2 (Axiom 2 of Theory 2): $\neg$(*has not magical powers*)(Fairy) = (*has magical powers*) (Fairy) and $AT_2$3 (Axiom 3 of Theory 2):

$$(\forall x)\left(woman(x) \wedge Mhas\ not\ magical\ powers \rightarrow has\ not\ magical\ powers(x)\right)$$

then $\exists Fairy \in X$; (*has not magical powers*) (Fairy) is not a theorem.

Default logic [19] can express facts like "*by default, something is true*"; by contrast, standard logic can only express that something is true or that something is false. This is a problem because reasoning often involves facts that are true in the majority of cases, but not always. A default theory is a pair $\langle W, D \rangle$, where W is a set of logical formulae, called *the background theory*, that formalize the facts that are known for sure and D is a set of *default rules*, each one being of the form:

$$\frac{Prerequisite: \ Justification_1, \ldots, Justification_n}{Conclusion}$$

According to this default, if we believe that a prerequisite is true, and each justification is true$_i$ then it is consistent with our current beliefs, so we are led to believe that the conclusion is true. The logical formulae in W and all default formulae were originally assumed to be first-order logic formulae, but they can potentially be formulae in an arbitrary formal logic.

Example 7. *The default rule "women have not magical powers" is formalized by the following default:*

$$D = \left\{ \frac{Woman(x) : has\ not\ magical\ powers(x)}{has\ not\ magical\ powers(x)} \right\}$$

This rule means that, if x is a woman, and it can be assumed that she has not magical powers, then we can conclude that she has not magical powers. A background theory containing some facts about woman is as follows:

$$W = \left\{ Woman(Farmer), Woman(Princess), has\ magical\ powers(Witch) \right\}$$

According to this default rule, a witch has not magical powers because the precondition Woman (Farmer) is true and the justification *has not magical powers* as (Farmer) is not inconsistent with what is currently known. On the contrary, Woman (Witch) does not allow concluding *has not magical powers* (Witch): Even if the precondition of the default Woman (Witch) is true, the justification *has not magical powers* as (Witch) is inconsistent with what is known. From this background theory and this default, Woman (Fairy) cannot be concluded because the default rule only allows deriving *has not magical powers (X)* from *Woman (X)*, but not vice versa. Deriving the antecedents of an inference rule from the consequences is a form of explanation of the consequences and is the aim of abductive reasoning. A common default assumption is that which is not known to be true is believed to be false. This is known as the Closed World Assumption and is formalized in default logic using a default like the following one for every fact F: $\frac{:\neg F}{\neg F}$. A default rule can be applied to a theory if its precondition is entailed by the theory and its justifications are all consistent with the theory. The application of a default rule leads to the addition of its consequence to the theory. Other default rules may then be applied to the resulting theory. When the theory is such that no other default can be applied, the theory is called an extension of the default theory. The default rules may be applied in a different order, and this may lead to different extensions.

Example 8. *The Fata Morgana example is a default theory with two extensions:*

$$\left\langle \begin{array}{l} \left\{ \frac{Woman(x):\neg has\ magical\ powers(x)}{\neg has\ magical\ powers(x)}, \frac{Fairy(x):has\ magical\ powers(x)}{has\ magical\ powers(x)} \right\}, \\ \{Woman(Morgana), Fairy(Morgana\} \end{array} \right\rangle$$

Since Morgana is both, a woman and a fairy, both defaults can be applied.

However, applying the first default leads to the conclusion that Morgana has not magical powers, which makes the second default not applicable. In the same way, applying the second default we obtain that Morgana has magical powers, thus making the first default not applicable. This particular default theory therefore has two extensions, one in which *has not magical powers* (Morgana) is true, and one in which *has not magical powers* (Morgana) is false.

A default $\frac{A:B_1,B_2,\ldots,B_n}{C}$ is applicable to a propositional theory T if $T = A$ and all theories $T \cup \{B_i\}$ are consistent. The application of this default to T leads to the theory $T \cup \{C_i\}$. In the Fata Morgana example, the application of the first default leads to a theory to which the second default cannot be applied and vice versa. As a result, two extensions are generated: One in which Morgana has magical powers and one in which Morgana has not magical powers. The final check of consistency of the justifications of all defaults that have been applied implies that some theories do not have any extensions. In particular, this happens whenever this check fails for every possible sequence of applicable defaults.

4. Some Interesing Problems

By way of example, we present some problems with the concept of existence.

4.1. First Problem

A particularly interesting problem arises with phrases like "*Sarah exists*", "*there are democracies*", and the like. How should we treat them from the formal point of view? We can treat them as "*Sarah*

formally exists", "*Some democracies formally exist*"; but that is not the only possibility, and often it is not very convenient to proceed in this way. It may be more advantageous to interpret the phrases listed as "*Sarah indeed exists*" and "*some democracies exist in reality*", in which we treat "*actually exist*", "*in fact exist*", "*To exist in the consciousness of man*", "*subjectively exist for me*", etc., as expressions of functions of the same type as "*to be professor*" or "*sleep*". When we do this, the different ways of not formally exist have no special symbolization, and do not differ in any way from other logical predicates, whilst for *not formally exist*, we must use a special symbolization, as that cannot be treated as a logical predicate. This procedure does not lead to contradictions. If we say, "*Fairies do not have effective existence*" using "*does not effectively exist*" as a representation of a function, this only means, for Theorem 1, "*Fairies formally exist*." The two sentences are perfectly compatible.

4.2. Second Problem

Let's take a case from the history of philosophy. We started with the investigation of Descartes famous phrase *Cogito, ergo sum*, considered as reasoning. Two possibilities of treatment are presented:

(1) I think (I fulfill the function of thinking), then I formally exist. Let T be "*to think*" and I be "*be me*". According to Theorem 1: $T|x(I(x)) \Rightarrow \exists_F x(I(x))$. We can accept the premise, but the conclusion does not say anything about effective existence. The universe of discourse of Philosophy is more extensive than that which effectively exists elsewhere.

(2) To obtain the conclusion, "*I really am*" one can use reasoning, e.g., thesingular syllogism Barbara:

All people who think effectively exists
I think
Then I effectively exist

In this case the result is consistent with the premises and should be accepted.

4.3. Third Problem: The Existence of God (Amselm's Ontological Argument)

The *ens perfectissimum* satisfies the function "*be taken by us in the idea*," then, and is satisfied by Theorem 1 formally. There is no need to resort to the highest perfection, but we cannot say anything about the effective existence. To obtain this last result we should have premises "*What is the most perfect in the Idea that has effective existence*" and "*God is the most perfect in the Idea*".

4.4. Fourth Problem

Let us suppose the phrase "*Fairies are a myth*". On the one hand it means that the fairies have no effective existence, while satisfying the function *to be a myth* to have formal existence, by Theorem 1. This contradiction is easily resolved by considering fairies included in the universe of discourse, distinguishing between formal existence and effective existence. Satisfy function "*be a myth*" implies non-effective existence (according to the idea expressed in the predicate) and formal existence (according to Theorem 1). Fairies have no effective existence while having formal existence. Let *My* be "*be a myth*", H be "*be a fairy*", and we have:

$$My|x(H(x)) \Rightarrow \exists_M x(H(x)) \Rightarrow \neg\exists_R|x(H(x))$$
$$My|x(H(x)) \Rightarrow \exists_M x(H(x)) \Rightarrow \exists_F|x(H(x))$$

4.5. Fifth Problem

The fifth and final problem is the old question of Plato of the existence of non-being or nothingness. We have seen that non-being (if identified with no formal existence) is not a function, and for this reason we cannot form the class of what is not. However, if the class of what is then is narrower than the universe of discourse used, then the class of what is not, is a class of individuals and formally exists. Moreover, in symbolic logic works with an empty class that does not contain elements that are

sometimes called "*nothing*." Nothingness in this sense formally exists, as has been indicated above. The empty or null set is a strange concept. It is a set, but it has no members. This is of course not a contingent fact about it, but one bound up with its very identity: The null set is *essentially* empty. Intuitively, however, one might have thought that a set is a group of two or more things. Indeed, Georg Cantor famously defines a set (*Menge*) as "*any collection into a whole (Zusammenfassung zu einem Ganzen) of definite and separate objects of our intuition and thought*." [20–22]. In the case of the null set, however, there are no definite objects that it collects. Thus, in what sense is the null set a set? Set theory can be done either naively or axiomatically. In the standard axiomatic approach to the subject, that of Zermelo-Fraenkel, the existence of the null set is posited in a special axiom. In Zermelo's 1908 formulation, both the null set and singleton sets are posited in his *Axiom der Elementarmengen*. About the null set Zermelo writes, "*There exists a (fictitious) set, the null set, 0 that contains no element at all*." ([23]. p. 202). One curious feature of this Zermelian formulation is that 'fictitious' appears to cancel out exists.' To *exist*, if it means anything, is to exist in reality, in splendid independence of language and mind. Something that exists as a fiction precisely does not exist. There is a more serious problem. Intuitively, the existence of a set depends on the existence of its members. It exists *because* the elements exist, not vice versa. In the case of the null set, however, there is nothing on which the null set can depend for its existence. Bertrand Russell refers to the difficulty in his early *Principles of Mathematics*. In Whitehead and Russell's *Principia Mathematica*, we learn that "*to say that a class exists is equivalent to saying that the class is not equal to the null-class*." [24]. It seems to follow from this that the null set does not exist. But two sets A and B are said to be disjoint if they have no members in common. What then is the intersection of two disjoint sets? The intersection of any two sets is a set. We also see the intersection of A' and B' (also disjoint) to be the same as the intersection of A and B. Thus, we speak of *the* null set, where 'the' connotes uniqueness. The Union Axiom states that, given any set A, there exists a set UA the members of which are exactly the members of the members of A. Now, suppose we apply the Union Axiom to the set $\{x_1, x_2\}$. Since the members of this set do not have members, $U\{x_1, x_2\}$ = the null set. In general, the application of the Union Axiom to any set the members of which are non-sets yields the null set. The uniqueness of the null set can be proven by *reductio ad absurdum*. In such a mode of proof one attempts to show that a certain assumption, in the presence of propositions antecedently accepted, implies a contradiction. Thus, assume that the null set is not unique: Assume that there are two null sets, A and A'. Then, by the Axiom of Extensionality (two sets are the same iff they all have the same members), A has a member that A' does not have, or vice versa. However, this result is in a contradiction inasmuch as neither A nor A' have a member. Therefore, the null set is unique. Quine notices that in New Foundations one can prove the existence of the empty set, the set containing the empty set, the set containing the set containing the empty set, and so on [25].

5. Conclusions

Formal existence, which coincides with belonging to the universe of discourse, is of great importance to formal logic. We can form various systems with different universes of discourse, which gives formal existence a relative character. This allows for fully rigorous delimitations, contrary to the vagueness of "exist" in the traditional or common use, which attributes to "*existence*" an absolute and universal character.

The relativization of existence is surprising only if an absolute character is attributed to the word "*exist*" in common language. Now, there are sentences that use "exist" in a sense as: "*This person does not exist for me*," where there is formed a universe of discourse that excludes certain individuals. Usually, however, the term "*exist*" is used with an absolute and general sense. However, at the moment we want to indicate, problems arise distinguishing between "*really exist*" and "*exist in the consciousness of man*," etc. Each of these terms is itself a source of new discussions, often characterized by darkness and confusion. For example, is there really an electromagnetic field and potential energy? In principle, the indicated relativization is not negative. It can choose strictly defined universes of discourse for each science, especially when that science is already rigorously formalized.

From a formal point of view, no universe of discourse is given in advance; any universe of discourse that satisfies the necessary conditions can be used. The extended epistemological belief that there is a universe of discourse defined rigorously, that would be the true and should be "*the universe of discourse of logic*", cannot be justified, as we have seen.

This philosophical and logical analysis of the problem of existence, by placing existence in an epistemological rather than the usual metaphysical context is associated with a similar move in constructivism in psychology. It would be interesting to extend this logical-philosophical domain in an interdisciplinary way to unite this work with radical constructivism where metaphysics is explicitly rejected in favour of an epistemological grounding of knowledge [26].

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References

1. Russell, B. On Denoting. *Mind* **1905**, *14*, 479–493. [CrossRef]
2. Russell, B. *El Conocimiento Humano*; Trans: Nestor Míguez; Orbis-Hyspamérica: Madrid, España, 1984. (In Spanish)
3. Ogden, C.K.; Richard, I.A. *The Meaning of Meaning. With a New Introduction by Umberto Eco*; Harcourt Brace Jovanovich Publishers: Orlando, FL, USA, 1989.
4. Carnap, R. *Introduction to Semantics*; Harvard University Press: Cambridge, MA, USA, 1942.
5. Nescolarde-Selva, J.A.; Usó-Doménech, J.L.; Gash, H. A theorical point of view of reality, perception, and language. *Complexity* **2014**, *20*, 27–37. [CrossRef]
6. Nescolarde-Selva, J.A.; Usó-Doménech, J.L.; Gash, H. A logic-mathematic point of view of reality, perception, and language. *Complexity* **2014**, *20*, 58–67. [CrossRef]
7. Nescolarde-Selva, J.; Usó-Doménech, J.L.; Gash, H. Language, Values, and Ideology in Complex Human Societies. *Cybern. Syst. Int. J.* **2015**, *46*, 390–422. [CrossRef]
8. Corcoran, J. *Universe of Discourse*; Cambridge Dictionary of Philosophy; Cambridge University Press: Cambridge, UK, 1995.
9. Bueno, O.; Shalkowski, S.A. Logical Constants: A Modalist Approach. *Noûs* **2013**, *47*, 1–24. [CrossRef]
10. Meinong, A. On Object Theory. In *Realism and the Background of Phenomenology*; Chisholm, R., Ed.; The Free Press: Glencoe, Scotland, 1960.
11. Lambert, K. *Meinong and the Principle of Independence. Its Place in Meinong's Theory of Objects and Its Significance in Contemporary Philosophical Logic*; Cambridge University Press: Cambridge, UK, 1983.
12. Marcus, R.B. A Functional Calculus of First Order Based on Strict Implication. *J. Symb. Log.* **1946**, *11*, 1–16.
13. Linsky, B.; Zalta, E. In Defense of the Simplest Quantified Modal Logic. In *Philosophical Perspectives 8: Logic and Language*; Tomberlin, J., Ed.; Ridgeview: Atascadero, CA, USA, 1994; pp. 431–458.
14. Brewka, G. *Nonmonotonic Reasoning: Logical Foundations of Commonsense*; Cambridge University Press: Cambridge, UK, 1991.
15. Brewka, G.; Dix, J.; Konolige, K. *Nonmonotonic Reasoning—An Overview*; CSLI Publications: Stanford, CA, USA, 1997.
16. Lukaszewicz, W. *Non-Monotonic Reasoning*; Ellis-Horwood: Chichester, UK, 1990.
17. Marek, W.; Truszczynski, M. *Nonmonotonic Logics: Context-Dependent Reasoning*; Springer: Berlin, Germany, 2013.
18. Moore, R.C. Semantical considerations on nonmonotonic logic. *Artif. Intell.* **1985**, *25*, 75–94. [CrossRef]
19. Reiter, R. A logic for default reasoning. *Artif. Intell.* **1980**, *13*, 81–132. [CrossRef]
20. Cantor, G. *Contributions to the Founding of the Theory of Transfinite Numbers*; Dover Publications: Mineola, NY, USA, 1955.
21. Usó-Doménech, J.L.; Nescolarde-Selva, J.; Belmonte-Requena, M. Mathematics, philosophic and semantical considerations on infinity (I): General concepts. *Found. Sci.* **2016**, *21*, 615–630. [CrossRef]

22. Usó-Doménech, J.L.; Nescolarde-Selva, J.A.; Belmonte-Requena, M.; Gash, H. Walking through Cantor's Paradise and Escher's Garden: Epistemological Reflections on the Mathematical Infinite (I). *Cybern. Syst. Int. J.* **2015**, *46*, 423–437. [CrossRef]
23. Van Heijenoort, J. (Ed.) *From Frege to Gödel*; Harvard University Press: Cambridge, MA, USA, 1967.
24. Whitehead, A.N.; Russell, B. *Principia Mathematica*; Cambridge Mathematical Library; Cambridge University Press: Cambridge, UK, 1997.
25. Specker, E.P. The Axiom of Choice in Quine's New Foundations for Mathematical Logic. *Proc. Natl. Acad. Sci. USA* **1953**, *39*, 972–975. [CrossRef] [PubMed]
26. Glasersfeld, E. *The Construction of Knowledge*; Intersystems: Seaside, CA, USA, 1987.